Chapter 24

Basic probability

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

24.1 Outcomes events and sample spaces

An outcome is one possible result of a trial. An event is a collection of outcomes. A sample space lists all possible outcomes once each. A probability model must reflect whether outcomes are equally likely. Words such as certain, likely and impossible describe the chance of an event.

Worked example For a fair six sided die, the sample space is {1, 2, 3, 4, 5, 6}. The event of rolling an even number is {2, 4, 6}. All six single outcomes have equal probability under the fair die model.

24.2 Calculating probability

For equally likely outcomes, probability = number of favourable outcomes divided by total outcomes. Probabilities lie from 0 to 1 inclusive. Zero means impossible and 1 means certain. Give fractions in simplest form, or an equivalent decimal or percentage when requested.

Worked example A bag holds 3 red and 5 blue counters, all equally likely to be selected. P(red) = $\frac{3}{8}$ and P(blue) = $\frac{5}{8}$. The probability of selecting a green counter is 0.

24.3 Complements and mutually exclusive events

The complement of an event means it does not happen. P(not A) = 1 − P(A). Mutually exclusive events cannot occur together in a single trial, so their probabilities can be added. For overlapping events, subtract the overlap to avoid counting it twice.

Worked example On a fair die, P(not 6) = $1 - \frac{1}{6} = \frac{5}{6}$. P(1 or 2) = $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$. For even or greater than 4, favourable outcomes are 2, 4, 5 and 6, giving $\frac{2}{3}$.

24.4 Experimental probability

Relative frequency = observed successes divided by number of trials. It estimates a probability from data. More trials usually make the estimate more stable, but no fixed result is guaranteed. Expected frequency = probability × number of trials and describes a long run expectation.

Worked example A spinner lands on red 46 times in 200 trials. Estimated P(red) = $\frac{46}{200}$ = 0.23. If this estimate is used for 500 trials, the expected red count is 0.23 × 500 = 115.

24.5 Combined independent events

Independent events do not change each other's probabilities. Multiply probabilities along a sequence of independent events. List outcomes systematically using a table or tree when helpful. For draws with replacement, the bag contents return to the original state before the next draw.

Worked example Two fair coin tosses have equally likely outcomes HH, HT, TH and TT. P(two heads) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. P(exactly one head) = $\frac{2}{4} = \frac{1}{2}$.

24.6 Drawing without replacement

Without replacement, the first draw changes the available outcomes, so later probabilities must change. Multiply along each relevant branch and add probabilities of different exclusive routes. State assumptions about random selection.

Worked example A bag contains 3 red and 2 blue counters. Without replacement, P(two red) = $\frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$. With replacement it would be $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$.

Chapter 24 practice

Work in your notebook. Show your method and include units where needed.

1. Find P(rolling a prime) on a fair die.
2. If P(rain) = 0.35, find P(no rain).
3. A bag has 4 green and 6 yellow counters. Find P(green).
4. A fair coin is tossed 120 times. Find the expected number of heads.
5. Two counters are drawn without replacement from 4 red and 1 blue. Find P(two red).

Answers with methods

1. Prime outcomes are 2, 3 and 5. Probability = $\frac{3}{6} = \frac{1}{2}$.

2. $1 - 0.35 = 0.65$.

3. $\frac{4}{10} = \frac{2}{5}$.

4. $120 \times \frac{1}{2} = 60$. The actual number may differ.

5. $\frac{4}{5} \times \frac{3}{4} = \frac{3}{5}$.

Common mistake to avoid

Equally likely outcomes are required for simple counting probability. A spinner with unequal sectors does not give every colour the same chance.