Chapter 21

Perimeter area surface area and volume

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

21.1 Perimeter and composite boundaries

Perimeter is the total distance around the outside of a flat shape. Add all boundary lengths in compatible units. A rectangle has perimeter 2(length + width). Find missing sides of composite shapes from equal horizontal or vertical totals. Do not count internal joining lines.

Worked example A rectangle measuring 8 cm by 5 cm has perimeter 2(8 + 5) = 26 cm. If a 2 cm square is cut from one corner, two removed boundary lengths are replaced by two equal new lengths, so the perimeter remains 26 cm.

21.2 Areas of common plane shapes

Area measures the surface inside a boundary and uses square units. Rectangle area is length × width. Triangle area is half × base × perpendicular height. Parallelogram area is base × perpendicular height. Trapezium area is half × sum of parallel sides × perpendicular height.

Worked example A triangle with base 12 cm and perpendicular height 7 cm has area $\frac{1}{2} \times 12 \times 7 = 42$ cm². A trapezium with parallel sides 6 cm and 10 cm and height 4 cm has area $\frac{1}{2} \times (6 + 10) \times 4 = 32$ cm².

21.3 Circles and semicircles

For radius r and diameter d, circumference = 2πr = πd and area = πr². Keep π exact until a rounded answer is required, unless a question gives a value for π. A semicircle has half a circle area, but its perimeter also includes its diameter.

Worked example A semicircle of radius 7 cm has area $\frac{49\pi}{2}$ cm². Its perimeter is $7\pi + 14$ cm. Using $\pi \approx 3.14159$ gives area about 76.97 cm² and perimeter about 35.99 cm.

21.4 Composite area

Split a composite shape into familiar nonoverlapping parts and add their areas. Alternatively, find the area of an enclosing shape and subtract the missing regions. Mark which measurements belong to each part. Areas can be added only when expressed in the same square units.

Worked example An L shape fits inside a 10 cm by 8 cm rectangle with a 4 cm by 3 cm corner missing. Its area is $80 - 12 = 68$ cm². Adding the side lengths would give a perimeter calculation, not an area.

21.5 Surface area

Surface area is the sum of all exposed surfaces of a solid. A cuboid has area $2(lw + lh + wh)$; a cube has area $6s^2$. A closed cylinder has area $2\pi r^2 + 2\pi rh$. An open container may have a missing top, so include only the surfaces present.

Worked example A closed cuboid of length 6 cm, width 4 cm and height 3 cm has surface area $2(24 + 18 + 12) = 108$ cm². Without its 6 cm by 4 cm top, the surface area is $108 - 24 = 84$ cm².

21.6 Volume and capacity

Volume uses cubic units. A cuboid has volume lwh and a cube has volume s^3. A prism has volume cross sectional area × perpendicular length. A cylinder has volume $\pi r^2 h$. Convert volume to capacity using 1 cm³ = 1 mL and 1,000 cm³ = 1 L.

Worked example A triangular prism has cross section area 18 cm² and length 10 cm, so its volume is 180 cm³. A tank measuring 40 cm by 25 cm by 30 cm holds 30,000 cm³ = 30 L when full.

Chapter 21 practice

Work in your notebook. Show your method and include units where needed.

1. Find the area and perimeter of a 9 cm by 4 cm rectangle.
2. Find the area of a triangle with base 15 m and height 8 m.
3. Find the circumference of a circle of radius 6 cm in terms of π.
4. Find the surface area of a cube of side 5 cm.
5. A cuboid is 12 cm by 5 cm by 4 cm. Find its volume.

Answers with methods

1. Area = $9 \times 4 = 36$ cm². Perimeter = $2(9 + 4) = 26$ cm.

2. $\frac{1}{2} \times 15 \times 8 = 60$ m².

3. $2\pi \times 6 = 12\pi$ cm.

4. $6 \times 5^2 = 150$ cm².

5. $12 \times 5 \times 4 = 240$ cm³.

Common mistake to avoid

Surface area uses square units; volume uses cubic units. The sloping side of a triangle is not its perpendicular height unless it is perpendicular to the base.