Chapter 19

Number patterns and sequences

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

19.1 Recognising a sequence

A sequence is an ordered list of terms. Each term has a position, beginning with position 1 unless stated otherwise. Look for a rule involving differences, ratios, alternating operations or known number families. A few terms can fit many rules, so describe the intended rule clearly.

Worked example The sequence 5, 9, 13, 17 has constant difference 4. Its fifth term is 21. The number 5 is a term value, while 1 is its position.

19.2 Arithmetic sequences

An arithmetic sequence has a constant difference d. Its nth term is first term + (n − 1)d. Simplify this expression when useful. The rule gives any term directly without listing all earlier terms. A negative common difference gives a decreasing sequence.

Worked example For 7, 10, 13, 16, the first term is 7 and d = 3. The nth term is $7 + 3(n - 1) = 3n + 4$. The twentieth term is $3 \times 20 + 4 = 64$.

19.3 Geometric sequences

A geometric sequence has a constant multiplying factor, called the common ratio. Its nth term is first term × ratio raised to the power n − 1. The ratio can be a fraction or negative. Check the ratio rather than assuming every growing pattern is arithmetic.

Worked example The sequence 3, 6, 12, 24 has ratio 2. Its nth term is $3 \times 2^{n-1}$. The sixth term is $3 \times 2^5 = 96$. The sequence 80, 40, 20, 10 has ratio $\frac{1}{2}$.

19.4 Square cubic triangular and recursive patterns

Square numbers are 1, 4, 9, 16 and so on. Cubes begin 1, 8, 27, 64. Triangular numbers begin 1, 3, 6, 10 and add consecutive counting numbers. A recursive rule defines a term using earlier terms, so starting values must be given.

Worked example A sequence beginning 1, 1 and adding the previous two terms continues 2, 3, 5, 8, 13. Triangular numbers have nth term $\frac{n(n+1)}{2}$, so the sixth is $\frac{6 \times 7}{2} = 21$.

19.5 Testing whether a value is a term

Set the nth term expression equal to the proposed value and solve for n. A term exists only if the resulting position is a positive integer under the stated indexing. Use a pattern rule to explain rather than relying on a long list.

Worked example Is 100 in the sequence with nth term 4n + 1? Solve 4n + 1 = 100 to get n = 24.75, so no. For 101, n = 25, so it is the twenty fifth term.

Chapter 19 practice

Work in your notebook. Show your method and include units where needed.

1. Continue 18, 14, 10, 6 for two terms.
2. Find the nth term of 6, 11, 16, 21.
3. Find the tenth term of 2n + 7.
4. Continue 81, 27, 9, 3 for two terms.
5. Is 62 in the sequence 5n + 2?

Answers with methods

1. The difference is −4, so the next terms are 2 and −2.

2. 6 + 5(n − 1) = 5n + 1.

3. 2 × 10 + 7 = 27.

4. Divide by 3 each time, giving 1 and $\frac{1}{3}$.

5. 5n + 2 = 62 gives n = 12, so yes, it is the twelfth term.

Common mistake to avoid

The common difference is not usually the whole nth term rule. For 5, 8, 11, the rule is 3n + 2, not just 3n.