Chapter 14

Squares cubes roots and indices

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

14.1 Squares and square roots

The square of a number is its product with itself. The principal square root of a nonnegative number is the nonnegative number whose square gives it. Memorise small square numbers to recognise them quickly. An equation such as $x^2 = 25$ has two real solutions even though $\sqrt{25}$ has one principal value.

Worked example $7^2 = 49$ and $\sqrt{49} = 7$. The solutions to $x^2 = 49$ are $x = 7$ and $x = -7$ because both numbers have square 49.

14.2 Cubes and cube roots

A cube multiplies a number by itself three times. The cube root reverses cubing. A negative number has a negative cube and a negative real cube root. Square units measure area; cubic units measure volume.

Worked example $4^3 = 64$, so $\sqrt[3]{64} = 4$. Also, $(-3)^3 = -27$ and $\sqrt[3]{(-27)} = -3$. A cube of side 4 cm has volume 64 cm^3.

14.3 Products and quotients of powers

An index tells how often the base appears as a factor. For the same nonzero base, multiply powers by adding indices and divide by subtracting indices. The bases must match. These rules describe products and quotients, not sums.

Worked example $2^3 \times 2^4 = 2^7 = 128$. Also, $5^6 \div 5^2 = 5^4 = 625$. But $2^3 + 2^4 = 8 + 16 = 24$, not 2^7.

14.4 Powers of powers and products

Raise a power to another power by multiplying the indices. A power of a product applies to each factor. A power of a quotient applies to numerator and denominator, provided the denominator is not zero. Brackets define the base.

Worked example $(3^2)^3 = 3^6 = 729$. Also, $(2 \times 5)^2 = 2^2 \times 5^2 = 100$, and $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$.

14.5 Zero negative and fractional indices

For a nonzero base a, $a^0 = 1$ and a raised to a negative integer is the reciprocal of the corresponding positive power. For positive a, a to the power one half means $\sqrt{a}$ and one third means $\sqrt[3]{a}$. Zero to a negative power is undefined.

Worked example $7^0 = 1$. Also, $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$. The value 16 raised to the power $\frac{1}{2}$ is $\sqrt{16} = 4$. The expression 0^0 is not assigned a value in this course.

Chapter 14 practice

Work in your notebook. Show your method and include units where needed.

1. Calculate 13^2.
2. Find $\sqrt[3]{125}$.
3. Simplify $3^2 \times 3^5$ as one power.
4. Evaluate 10^{-2}.
5. Solve $x^2 = 36$.

Answers with methods

1. $13 \times 13 = 169$.

2. 5, because $5^3 = 125$.

3. 3^7, because $2 + 5 = 7$.

4. $\frac{1}{10^2} = \frac{1}{100} = 0.01$.

5. $x = 6$ or $x = -6$. Both square to 36.

Common mistake to avoid

$\sqrt{(a + b)}$ is generally not $\sqrt{a} + \sqrt{b}$. For example, $\sqrt{(9 + 16)} = 5$, but $\sqrt{9} + \sqrt{16} = 7$.