Chapter 12

Ratio proportion and the unitary method

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

12.1 Ratio notation and simplification

A ratio compares amounts in a stated order. Write quantities in the same units before simplifying. Divide every part by the same common factor. A ratio compares parts with parts; a fraction can compare one part with the total. A ratio need not add to 1 or 100.

Worked example Red to blue beads is 12 : 18 = 2 : 3. There are 5 parts in total, so the fraction that is red is $\frac{2}{5}$, not $\frac{2}{3}$. Also, 1 m : 25 cm = 100 : 25 = 4 : 1.

12.2 Sharing in a ratio

Add the ratio parts, divide the total by that number, then multiply by each part. Check that the shares add to the original amount. For a difference problem, compare the difference in parts with the given difference in amounts.

Worked example Share 360 in the ratio 2 : 3 : 4. There are 9 parts, so each part is 40. The shares are 80, 120 and 160. Their sum is 360.

12.3 The unitary method

Find the value of one unit and then scale to the required number. This method works when the rate is constant. Include units at each step to avoid reversing the calculation. The same method applies to prices, recipes, wages and distances at constant speed.

Worked example Five notebooks cost 175 rupees. One notebook costs 175 ÷ 5 = 35 rupees. Eight notebooks cost 8 × 35 = 280 rupees, assuming the price per notebook stays the same.

12.4 Direct proportion

Two quantities are directly proportional when their ratio remains constant. Multiplying one by a factor multiplies the other by the same factor. Write $y = kx$, where k is the constant of proportionality. A direct proportion graph is a straight line through the origin.

Worked example A recipe uses 180 g flour for 6 cakes. For 15 cakes, the scale factor is $15 \div 6 = 2.5$, so flour needed is $180 \times 2.5 = 450$ g.

12.5 Inverse proportion

Two positive quantities are inversely proportional when their product remains constant. Increasing one decreases the other by the same factor. This model requires unchanged total work and a constant rate per worker or machine. It does not apply automatically to every real situation.

Worked example Four equally efficient machines finish a job in 9 hours. The work is $4 \times 9 = 36$ machine hours. Six such machines take $36 \div 6 = 6$ hours, assuming they can work together without delays.

12.6 Scale drawings

A scale compares a drawing length with the corresponding real length. Convert both to the same units. A scale of $1 : n$ means 1 unit on the drawing represents n of the same units in reality. The scale factor for area is n^2 and for volume is n^3.

Worked example On a $1 : 50{,}000$ map, 3 cm represents 150,000 cm = 1.5 km. A real distance of 2 km is 200,000 cm, so it measures 4 cm on this map.

Chapter 12 practice

Work in your notebook. Show your method and include units where needed.

1. Simplify 45 : 60.

2. Share 420 rupees in the ratio 3 : 4.

3. Three kilograms of rice cost 156 rupees. Find the cost of 5 kg.

4. Eight workers need 15 days. How long do 12 equally efficient workers need?

5. On a 1 : 20,000 map, a road is 7 cm long. Find its real length.

Answers with methods

1. Divide both terms by 15 to obtain 3 : 4.

2. 7 parts give 60 rupees per part. The shares are 180 and 240 rupees.

3. One kilogram costs 52 rupees, so 5 kg cost 260 rupees.

4. 8 × 15 = 120 worker days. Time = 120 ÷ 12 = 10 days, assuming constant efficiency and divisible work.

5. 7 × 20,000 = 140,000 cm = 1.4 km.

Common mistake to avoid

Inverse proportion uses a constant product. Direct proportion uses a constant ratio. Decide which relationship the context supports before calculating.