Chapter 8

Divisibility prime factorisation HCF and LCM

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

8.1 Divisibility rules

A number is divisible by 2 if its last digit is even; by 5 if it ends in 0 or 5; and by 10 if it ends in 0. It is divisible by 3 or 9 when its digit sum is divisible by 3 or 9. Divisibility by 6 requires divisibility by both 2 and 3.

Worked example For 738, the digit sum is 18. It is divisible by 3 and 9. It ends in an even digit, so it is divisible by 2 and therefore by 6. It does not end in 0 or 5, so it is not divisible by 5.

8.2 Further divisibility tests

A number is divisible by 4 if its last two digits form a multiple of 4. For 8, test the last three digits. For 11, the difference between alternating digit sums must be zero or a multiple of 11. These tests avoid full division.

Worked example 3,216 is divisible by 4 because 16 is, and by 8 because $216 \div 8 = 27$. For 2,728, $(2 + 2) - (7 + 8) = -11$, so 2,728 is divisible by 11.

8.3 Prime factorisation

Prime factorisation expresses a whole number greater than 1 as a product of primes. Use a factor tree or repeated division by primes. Continue until every factor is prime, then collect repeated factors as powers. Apart from order, the prime factorisation is unique.

Worked example $360 = 36 \times 10 = 2 \times 2 \times 3 \times 3 \times 2 \times 5 = 2^3 \times 3^2 \times 5$. Multiplying the factors back gives $8 \times 9 \times 5 = 360$.

8.4 Highest common factor

The HCF is the greatest positive number dividing every given number. In prime factorisations, use only primes common to all numbers with the smallest power present in each. HCF is also called greatest common divisor. It helps divide different quantities into the largest number of identical packs.

Worked example $48 = 2^4 \times 3$ and $72 = 2^3 \times 3^2$. The HCF is $2^3 \times 3 = 24$. Forty eight pencils and 72 pens make 24 identical packs with 2 pencils and 3 pens each.

8.5 Lowest common multiple

The LCM is the smallest positive common multiple. Use every prime appearing in any factorisation with its largest power. LCM helps synchronise repeating events and find common denominators. For two positive integers, HCF × LCM equals their product.

Worked example $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$, so LCM $= 2^2 \times 3^2 = 36$. Bells ringing every 12 and 18 minutes ring together again after 36 minutes if they start together.

Chapter 8 practice

Work in your notebook. Show your method and include units where needed.

1. Is 4,572 divisible by 9?
2. Find the prime factorisation of 84.
3. Find the HCF of 30 and 42.
4. Find the LCM of 8 and 12.
5. Two lights flash every 15 and 20 seconds. When will they next flash together?

Answers with methods

1. Its digit sum is 18, which is divisible by 9, so yes.

2. $84 = 2 \times 42 = 2 \times 2 \times 21 = 2^2 \times 3 \times 7$.

3. $30 = 2 \times 3 \times 5$ and $42 = 2 \times 3 \times 7$, so HCF = 6.

4. $8 = 2^3$ and $12 = 2^2 \times 3$, so LCM $= 2^3 \times 3 = 24$.

5. LCM of 15 and 20 is 60, so after 60 seconds.

Common mistake to avoid

Multiplying two numbers always gives a common multiple, but it gives the LCM only when the numbers have no common factor greater than 1.