Basic Foundation Mathematics

Complete Chapter Notes

Sriman Maths Academy

A learning book for all grades building a secure foundation in mathematics. The 25 chapters follow the Topics list in the Basic Foundation Mathematics course.

Begin with the chapters that match your current understanding. Later chapters extend the same foundations into algebra, geometry, graphs, data and probability.

How to use this book

Read each topic carefully and copy the worked example one step at a time.

Cover the answers while attempting the five practice questions at the end of each chapter.

Check your method as well as your final answer. Revisit the relevant topic whenever a step is unclear.

Use the separate chapter files for focused study. Keep this complete book for revision across topics.

All practice tax and currency conversion rates are illustrative. Use the rates supplied in each question.

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Chapter 1

Number sense counting and place value

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

1.1 Counting and grouping

Counting tells us how many objects are in a collection. Match each object to one number word and count every object once. The final number gives the total. Count forwards to add objects and backwards to remove them. Grouping in twos, fives or tens makes a large collection easier to count.

Worked example There are 6 groups of 10 counters and 4 single counters. The total is 60 + 4 = 64. Counting by tens gives 10, 20, 30, 40, 50, 60, followed by 61, 62, 63, 64.

1.2 Digits and place value

The ten digits are 0 to 9. A digit has a face value and a place value. In a base ten system, each place is ten times the place to its right. Zero holds an empty place. From the right, whole number places are ones, tens, hundreds, thousands, ten thousands, hundred thousands and millions.

Worked example In 407,253, the digit 7 represents 7,000. Expanded form is 400,000 + 7,000 + 200 + 50 + 3. The zero shows that there are no ten thousands.

1.3 Reading and writing large numbers

Separate digits into groups to read large numbers. International grouping uses groups of three digits from the right. Indian grouping uses three digits at the right and then groups of two. The value stays the same whichever grouping system you use.

Worked example 1,234,567 is one million two hundred and thirty four thousand five hundred and sixty seven. The same number is written 12,34,567 in Indian grouping and read as twelve lakh thirty four thousand five hundred and sixty seven.

1.4 Decimal place value

The decimal point separates whole parts from parts smaller than one. The first three places to its right are tenths, hundredths and thousandths. Each step to the right divides the value by ten. Zeros at the end of a decimal do not change its value.

Worked example $32.405 = 30 + 2 + \frac{4}{10} + \frac{0}{100} + \frac{5}{1000}$. The 5 represents five thousandths. Also, 3.5 = 3.50 because both describe the same amount.

1.5 Partitioning and regrouping

Partitioning breaks a number into useful parts. Regrouping exchanges one unit for ten smaller units without changing the total. These ideas explain carrying in addition and exchanging in subtraction. A number can have several correct partitions.

Worked example 326 can be 300 + 20 + 6 or 200 + 120 + 6. For subtraction, 52 can be regrouped as 4 tens and 12 ones. Both representations still equal 52.

Chapter 1 practice

Work in your notebook. Show your method and include units where needed.

1. Write the value of 8 in 582,416.
2. Write 60,000 + 900 + 7 as one number.
3. Write 4.072 in expanded form.
4. How many counters are in 9 groups of 10 and 6 singles?
5. Regroup 71 as tens and ones so that there are 11 ones.

Answers with methods

1. 8 is in the ten thousands place, so its value is 80,000.

2. 60,907. Zeros hold the thousands and tens places.

3. $4 + \frac{7}{100} + \frac{2}{1000}$.

4. 90 + 6 = 96 counters.

5. 6 tens and 11 ones, because 60 + 11 = 71.

Common mistake to avoid

A zero inside a number cannot usually be removed. 3,042 and 342 have different values. Say the place name before deciding the value of a digit.

Chapter 2

Types of numbers

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

2.1 Natural numbers and whole numbers

In this course, natural numbers are the counting numbers 1, 2, 3 and so on. Whole numbers include zero as well. Some books include zero among the natural numbers, so check the convention being used. A whole number has no fractional part.

Worked example 7 is both natural and whole. Zero is whole but is not natural under our convention. 2.5 is neither natural nor whole.

2.2 Integers and opposite numbers

Integers are negative whole numbers, zero and positive whole numbers. Every positive integer has a negative opposite equally far from zero. Zero is neither positive nor negative. Absolute value means distance from zero, so it is never negative.

Worked example The opposite of 8 is −8. Both are 8 units from zero, so $|8| = 8$ and $|-8| = 8$. A temperature of −3°C is represented by an integer.

2.3 Rational numbers

A rational number can be written as $\frac{p}{q}$, where p and q are integers and q is not zero. Every integer is rational because it can be written over 1. Terminating decimals and repeating decimals are rational. A fraction is undefined when its denominator is zero.

Worked example $5 = \frac{5}{1}$, $0.75 = \frac{3}{4}$, and $0.333\ldots = \frac{1}{3}$. All three are rational, although their written forms look different.

2.4 Irrational and real numbers

An irrational number cannot be written as a ratio of two integers. Its decimal continues forever without a repeating pattern. Examples include π and √2. Rational and irrational numbers together form the real numbers. A square root of a perfect square is rational.

Worked example √49 = 7 is rational. √50 is irrational because 50 is not a perfect square. The finite decimal 3.14 is rational and is only an approximation to π.

2.5 Even odd prime and composite numbers

An even integer is divisible by 2. An odd integer is not. A prime is an integer greater than 1 with exactly two positive factors. A composite is an integer greater than 1 with more than two positive factors. The number 1 is neither prime nor composite.

Worked example 2 is the only even prime. 15 is odd and composite because its positive factors are 1, 3, 5 and 15. Zero is even because $0 = 2 \times 0$.

Chapter 2 practice

Work in your notebook. Show your method and include units where needed.

1. Classify −12 as integer, rational or irrational.
2. Is √81 rational? Explain.
3. Is 0.454545… rational?
4. List the primes between 10 and 20.
5. Which of 0, 1, 2 and 9 are composite?

Answers with methods

1. −12 is an integer and rational because $-12 = \frac{-12}{1}$. It is not irrational.

2. Yes. √81 = 9, and 9 can be written as $\frac{9}{1}$.

3. Yes. The block 45 repeats, so it is a repeating rational decimal.

4. 11, 13, 17 and 19. Each has exactly two positive factors.

5. Only 9 is composite. Zero and 1 are neither prime nor composite; 2 is prime.

Common mistake to avoid

A decimal that never ends is not automatically irrational. A repeating decimal such as 0.121212… is rational.

Chapter 3

Comparing ordering and the number line

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

3.1 Comparison symbols

Use = for equal to, < for less than and > for greater than. The open side of an inequality faces the larger value. Compare whole numbers by the number of digits first, then compare from the left. Leading zeros do not make a number larger.

Worked example 9,087 > 9,078 because the thousands and hundreds agree, but 8 tens is greater than 7 tens. Also, 0064 = 64.

3.2 Ordering decimals

Align decimal points and compare corresponding places. You may add zeros at the right of a decimal without changing its value. Compare the whole parts first, then tenths, hundredths and further places until a difference appears.

Worked example Compare 4.7 and 4.68 by writing 4.70 and 4.68. Seven tenths exceeds six tenths, so 4.7 > 4.68. Ascending order is 4.608, 4.68, 4.7.

3.3 The number line

A number line has equally spaced marks for equal changes in value. Numbers increase to the right and decrease to the left. Choose a useful scale and label zero when it lies in the displayed interval. Fractions and decimals can be placed between integers.

Worked example If the distance from 0 to 1 is divided into four equal intervals, the marks are 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1. Each interval represents 0.25.

3.4 Comparing negative numbers

A number farther right on the number line is greater. Among negative numbers, the one closer to zero is greater. Think about temperature or debt to understand the order. Absolute size and numerical order answer different questions.

Worked example $-2 > -7$ because -2 is farther right. However, $|-7| > |-2|$ because the distance 7 is larger than the distance 2. Ascending order of -1, -6, 3 and 0 is -6, -1, 0, 3.

3.5 Comparing fractions and finding midpoints

Compare positive fractions using a common denominator, equivalent decimals or cross products when denominators are positive. For a midpoint, add the two values and divide by 2. A midpoint is equally far from both endpoints.

Worked example $\frac{3}{5} = \frac{24}{40}$ and $\frac{5}{8} = \frac{25}{40}$, so $\frac{3}{5} < \frac{5}{8}$. An integer denominator method gives $3 \times 8 = 24 < 25 = 5 \times 5$. The midpoint of -4 and 6 is $(-4 + 6) \div 2 = 1$.

Chapter 3 practice

Work in your notebook. Show your method and include units where needed.

1. Put 0.72, 0.7 and 0.702 in ascending order.
2. Insert < or > between −9 and −4.
3. Which is larger, $\frac{7}{12}$ or $\frac{3}{5}$?
4. Find the midpoint of 2.4 and 5.8.
5. A line runs from 0 to 2 in eight equal intervals. What is each interval?

Answers with methods

1. 0.700 < 0.702 < 0.720, so the order is 0.7, 0.702, 0.72.

2. −9 < −4 because −9 lies farther left.

3. $\frac{7}{12} = \frac{35}{60}$ and $\frac{3}{5} = \frac{36}{60}$, so $\frac{3}{5}$ is larger.

4. (2.4 + 5.8) ÷ 2 = 8.2 ÷ 2 = 4.1.

5. $2 \div 8 = \frac{1}{4} = 0.25$.

Common mistake to avoid

More decimal digits do not mean a larger value. 0.9 is greater than 0.125. Always compare place values.

Chapter 4

The four operations

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

4.1 Addition and its properties

Addition combines amounts. Align matching places in column addition and regroup whenever a place totals ten or more. Addition is commutative and associative, so you can change the order or grouping. Adding zero leaves a number unchanged.

Worked example 2,786 + 1,459: ones give 15, so write 5 and carry 1. Tens give 14, hundreds give 12 and thousands give 4. The total is 4,245. Check by estimating 2,800 + 1,500 ≈ 4,300.

4.2 Subtraction and exchanging

Subtraction finds what remains, a difference or a missing amount. Align places and exchange from a higher place when needed. Subtraction is the inverse of addition. Changing the order usually changes the result, so subtraction is not commutative.

Worked example 5,003 − 1,786 = 3,217. Regroup 5,003 as 4 thousands, 9 hundreds, 9 tens and 13 ones. Check: 3,217 + 1,786 = 5,003.

4.3 Multiplication and the distributive property

Multiplication describes equal groups, repeated addition and scaling. Multiply each place and add partial products. The distributive property splits a product into easier products. Multiplication is commutative and associative, and multiplying by 1 keeps the value.

Worked example 236 × 24 = 236 × (20 + 4) = 4,720 + 944 = 5,664. Estimate 240 × 24 ≈ 5,760 to check the scale of the answer.

4.4 Division and remainders

Division shares into equal groups or finds how many groups fit. Dividend = divisor × quotient + remainder. For positive whole number division, the remainder is nonnegative and smaller than the divisor. Division by zero is undefined.

Worked example 157 ÷ 6 = 26 remainder 1, because 6 × 26 + 1 = 157. If 157 children need buses holding 6 children each, 27 buses are needed because the remaining child needs another bus.

4.5 Decimal calculations and inverse checks

Align decimal points when adding or subtracting. For multiplication, multiply as whole numbers then use the total number of decimal places. For division, multiply dividend and divisor by the same power of ten to make the divisor a whole number. Check using the inverse operation.

Worked example 3.6 × 0.4 = 1.44 because 36 × 4 = 144 and there are two decimal places in total. Also, 7.56 ÷ 0.6 = 75.6 ÷ 6 = 12.6. Check: 12.6 × 0.6 = 7.56.

Chapter 4 practice

Work in your notebook. Show your method and include units where needed.

1. Calculate 4,675 + 2,958.
2. Calculate 8,002 − 3,847.
3. Calculate 125 × 48 using a useful partition.
4. Share 245 stickers equally among 8 children.
5. Calculate 6.72 ÷ 0.8.

Answers with methods

1. The sum is 7,633. Check: 7,633 − 2,958 = 4,675.

2. The difference is 4,155. Check: 4,155 + 3,847 = 8,002.

3. 125 × (40 + 8) = 5,000 + 1,000 = 6,000.

4. 245 ÷ 8 = 30 remainder 5. Each gets 30 stickers, with 5 left over.

5. 67.2 ÷ 8 = 8.4. Check: 8.4 × 0.8 = 6.72.

Common mistake to avoid

A remainder must be interpreted. Sometimes round up for containers, sometimes report the remainder, and sometimes continue as a decimal.

Chapter 5

Positive and negative numbers

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

5.1 Directed quantities

A sign gives direction relative to a chosen zero. Positive and negative values can describe temperatures, heights, movements and balances. State what zero and each direction mean in the context. An increase is represented by a positive change and a decrease by a negative change.

Worked example A submarine at −18 m rises 7 m. Its new position is −18 + 7 = −11 m. It is still below sea level, but it is closer to zero.

5.2 Adding signed numbers

To add numbers with the same sign, add their absolute values and keep the sign. With different signs, subtract the smaller absolute value from the larger and keep the sign of the number with larger absolute value. Opposite numbers sum to zero.

Worked example −8 + (−5) = −13. For −8 + 5, subtract 5 from 8 and keep the negative sign, giving −3. Also, −8 + 8 = 0.

5.3 Subtracting signed numbers

Subtracting a number is the same as adding its opposite. Rewrite the subtraction first, then use addition rules. Brackets help separate the operation symbol from a negative sign. On a number line, subtracting a negative moves right.

Worked example 6 − (−4) = 6 + 4 = 10. Also, −3 − 7 = −3 + (−7) = −10, and −3 − (−7) = −3 + 7 = 4.

5.4 Multiplying and dividing signed numbers

For multiplication and division, equal signs give a positive result and different signs give a negative result. Work out the sign separately from the absolute values. A product with an even number of negative factors is positive; an odd number gives a negative product.

Worked example $(-6) \times (-4) = 24$, while $24 \div (-6) = -4$. The product $(-2) \times (-3) \times (-5) = -30$ because it has three negative factors.

5.5 Signs brackets and powers

A negative sign outside brackets applies to the whole bracket. Powers act on their stated base. Brackets determine whether the negative sign is part of that base. Write each stage clearly before combining terms.

Worked example $(-3)^2 = (-3) \times (-3) = 9$. In contrast, $-3^2 = -(3 \times 3) = -9$. Also, $-(5 - 8) = -(-3) = 3$.

Chapter 5 practice

Work in your notebook. Show your method and include units where needed.

1. Calculate −14 + 9.
2. Calculate −5 − (−12).
3. Calculate (−7) × 8.
4. Calculate (−48) ÷ (−6).
5. A temperature rises from −6°C to 4°C. Find the rise.

Answers with methods

1. The absolute values differ by 5, and −14 has the larger absolute value, so the answer is −5.

2. −5 + 12 = 7.

3. Different signs give a negative result: −56.

4. Equal signs give a positive result: 8.

5. 4 − (−6) = 10°C.

Common mistake to avoid

The rule about equal signs applies to multiplication and division. It does not mean that adding two negatives makes a positive.

Chapter 6

Order of operations and brackets

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

6.1 The operation hierarchy

Evaluate brackets first, then powers and roots, then multiplication and division, then addition and subtraction. Multiplication and division have equal priority and are done from left to right. Addition and subtraction also have equal priority and are done from left to right.

Worked example $18 - 6 \div 3 \times 2 = 18 - 2 \times 2 = 18 - 4 = 14$. Doing the subtraction first would change the intended calculation.

6.2 Brackets within brackets

Start with the innermost brackets and work outwards. Keep all remaining parts of the expression unchanged at each step. Square brackets and braces can group calculations in the same way as round brackets. Copy signs carefully.

Worked example $3 \times [12 - (5 + 3)] = 3 \times [12 - 8] = 3 \times 4 = 12$. Each line removes only one completed part.

6.3 Powers roots and negative values

A power or root is evaluated before multiplication, division, addition or subtraction unless brackets change the order. A power applies only to its base. A root symbol covers the expression shown underneath it.

Worked example $2 + 3^2 \times 4 = 2 + 9 \times 4 = 38$. Also, $\sqrt{(16 + 9)} = \sqrt{25} = 5$, whereas $\sqrt{16} + \sqrt{9} = 4 + 3 = 7$.

6.4 Fraction bars as grouping symbols

A fraction bar groups the whole numerator and the whole denominator. Simplify each separately before dividing. If another operation is outside the fraction, apply the normal hierarchy. The denominator must not become zero.

Worked example $\frac{8+4}{7-3} = \frac{12}{4} = 3$. The expression means $(8 + 4) \div (7 - 3)$, rather than $8 + 4 \div 7 - 3$.

6.5 Building and checking expressions

Translate the order described in words into an expression. Use brackets to show a sum or difference that must be completed before another operation. Estimate the likely size, then evaluate line by line. Equivalent expressions can have different appearances.

Worked example Three times the sum of 7 and 5 is $3 \times (7 + 5) = 36$. Three times 7, then add 5, is $3 \times 7 + 5 = 26$. The wording determines the brackets.

Chapter 6 practice

Work in your notebook. Show your method and include units where needed.

1. Calculate $9 + 4 \times 3$.
2. Calculate $(9 + 4) \times 3$.
3. Calculate $30 - 12 \div 3 + 2$.
4. Calculate $\dfrac{15 - 3}{2 + 4}$.
5. Write and evaluate five less than the square of 6.

Answers with methods

1. $9 + 12 = 21$.

2. $13 \times 3 = 39$.

3. $30 - 4 + 2 = 26 + 2 = 28$.

4. $\dfrac{12}{6} = 2$.

5. $6^2 - 5 = 36 - 5 = 31$.

Common mistake to avoid

In $24 \div 6 \times 2$, division comes first because it is leftmost. The result is 8, not 2.

Chapter 7

Multiplication tables factors and multiples

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

7.1 Multiplication facts and inverse facts

A multiplication table lists products of one fixed number with counting numbers. Learn facts accurately and connect each fact to division. Use known facts, doubling and the commutative property to recover a fact you forget. Zero times any number is zero.

Worked example From $7 \times 8 = 56$, we know $8 \times 7 = 56$, $56 \div 7 = 8$ and $56 \div 8 = 7$. To find 6×8, double $3 \times 8 = 24$ to obtain 48.

7.2 Building tables efficiently

The 2 table doubles, the 4 table doubles twice and the 8 table doubles three times. The 5 table is half the 10 table. For the 9 table, subtract one group from ten groups. Extend tables beyond memorised values using the distributive property.

Worked example $9 \times 7 = 10 \times 7 - 7 = 63$. Also, $12 \times 8 = 10 \times 8 + 2 \times 8 = 80 + 16 = 96$, and $7 \times 13 = 7 \times 10 + 7 \times 3 = 91$.

7.3 Factors and factor pairs

A positive factor divides a whole number exactly. Find factors in pairs by testing divisors systematically. You can stop looking for new pairs after reaching the square root. A perfect square has a repeated middle pair.

Worked example Factor pairs of 36 are 1×36, 2×18, 3×12, 4×9 and 6×6. Its positive factors are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

7.4 Multiples and common multiples

Multiples result from multiplying by integers. For elementary lists of positive multiples, start with one times the number. Common multiples appear in both lists. Every nonzero whole number has infinitely many positive multiples but only finitely many positive factors.

Worked example Positive multiples of 6 include 6, 12, 18, 24 and 30. Multiples of 8 include 8, 16, 24 and 32. Therefore, 24 is a common multiple of 6 and 8.

7.5 Common factors and useful applications

Common factors divide each of the given numbers exactly. Use them to make equal groups without leftovers. Multiples help with repeating events and common denominators. A factor is usually no greater than a positive number; its positive multiples are no smaller than it.

Worked example The common factors of 18 and 24 are 1, 2, 3 and 6. Eighteen red beads and 24 blue beads can make 6 identical packs, each with 3 red and 4 blue beads.

Chapter 7 practice

Work in your notebook. Show your method and include units where needed.

1. Calculate 8 × 7 and write a related division fact.
2. List all positive factors of 28.
3. Write the first five positive multiples of 9.
4. List the common factors of 12 and 20.
5. Find a common multiple of 5 and 7 below 50.

Answers with methods

1. 8 × 7 = 56, so 56 ÷ 8 = 7.

2. 1, 2, 4, 7, 14 and 28, from the pairs 1 × 28, 2 × 14 and 4 × 7.

3. 9, 18, 27, 36 and 45.

4. Factors common to both are 1, 2 and 4.

5. 35 = 5 × 7, so 35 is a common multiple.

Common mistake to avoid

A factor divides a number. A multiple is obtained by multiplying it. For example, 3 is a factor of 12 and 36 is a multiple of 12.

Chapter 8

Divisibility prime factorisation HCF and LCM

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

8.1 Divisibility rules

A number is divisible by 2 if its last digit is even; by 5 if it ends in 0 or 5; and by 10 if it ends in 0. It is divisible by 3 or 9 when its digit sum is divisible by 3 or 9. Divisibility by 6 requires divisibility by both 2 and 3.

Worked example For 738, the digit sum is 18. It is divisible by 3 and 9. It ends in an even digit, so it is divisible by 2 and therefore by 6. It does not end in 0 or 5, so it is not divisible by 5.

8.2 Further divisibility tests

A number is divisible by 4 if its last two digits form a multiple of 4. For 8, test the last three digits. For 11, the difference between alternating digit sums must be zero or a multiple of 11. These tests avoid full division.

Worked example 3,216 is divisible by 4 because 16 is, and by 8 because $216 \div 8 = 27$. For 2,728, $(2 + 2) - (7 + 8) = -11$, so 2,728 is divisible by 11.

8.3 Prime factorisation

Prime factorisation expresses a whole number greater than 1 as a product of primes. Use a factor tree or repeated division by primes. Continue until every factor is prime, then collect repeated factors as powers. Apart from order, the prime factorisation is unique.

Worked example $360 = 36 \times 10 = 2 \times 2 \times 3 \times 3 \times 2 \times 5 = 2^3 \times 3^2 \times 5$. Multiplying the factors back gives $8 \times 9 \times 5 = 360$.

8.4 Highest common factor

The HCF is the greatest positive number dividing every given number. In prime factorisations, use only primes common to all numbers with the smallest power present in each. HCF is also called greatest common divisor. It helps divide different quantities into the largest number of identical packs.

Worked example $48 = 2^4 \times 3$ and $72 = 2^3 \times 3^2$. The HCF is $2^3 \times 3 = 24$. Forty eight pencils and 72 pens make 24 identical packs with 2 pencils and 3 pens each.

8.5 Lowest common multiple

The LCM is the smallest positive common multiple. Use every prime appearing in any factorisation with its largest power. LCM helps synchronise repeating events and find common denominators. For two positive integers, HCF × LCM equals their product.

Worked example $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$, so LCM $= 2^2 \times 3^2 = 36$. Bells ringing every 12 and 18 minutes ring together again after 36 minutes if they start together.

Chapter 8 practice

Work in your notebook. Show your method and include units where needed.

1. Is 4,572 divisible by 9?
2. Find the prime factorisation of 84.
3. Find the HCF of 30 and 42.
4. Find the LCM of 8 and 12.
5. Two lights flash every 15 and 20 seconds. When will they next flash together?

Answers with methods

1. Its digit sum is 18, which is divisible by 9, so yes.

2. $84 = 2 \times 42 = 2 \times 2 \times 21 = 2^2 \times 3 \times 7$.

3. $30 = 2 \times 3 \times 5$ and $42 = 2 \times 3 \times 7$, so HCF = 6.

4. $8 = 2^3$ and $12 = 2^2 \times 3$, so LCM $= 2^3 \times 3 = 24$.

5. LCM of 15 and 20 is 60, so after 60 seconds.

Common mistake to avoid

Multiplying two numbers always gives a common multiple, but it gives the LCM only when the numbers have no common factor greater than 1.

Chapter 9

Fractions mixed numbers and operations

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

9.1 Fractions and their types

A fraction represents division or equal parts of a whole. The denominator tells the number of equal parts and must be nonzero. The numerator tells how many parts are counted. A positive proper fraction has numerator smaller than denominator; an improper fraction has numerator at least as large.

Worked example $\frac{3}{8}$ means three of eight equal parts. $\frac{11}{4}$ is improper and equals $2\ \frac{3}{4}$, since $11 \div 4 = 2$ remainder 3. Convert back using $2 \times 4 + 3 = 11$.

9.2 Equivalent fractions and simplest form

Multiply or divide numerator and denominator by the same nonzero number to preserve a fraction. To simplify fully, divide both by their HCF. Equivalent fractions represent the same point on a number line. Compare fractions by giving them a common positive denominator.

Worked example $\frac{18}{24} = \frac{3}{4}$ after dividing both parts by 6. Also, $\frac{2}{3} = \frac{10}{15}$ and $\frac{3}{5} = \frac{9}{15}$, so $\frac{2}{3} > \frac{3}{5}$.

9.3 Adding and subtracting fractions

Equal denominators describe the same size of part, so add or subtract numerators and keep the denominator. For different denominators, first find equivalent fractions with a common denominator, preferably the LCM. Simplify the final answer. Convert mixed numbers to improper fractions when helpful.

Worked example $2\frac{1}{3} - \frac{3}{4} = \frac{7}{3} - \frac{3}{4} = \frac{28}{12} - \frac{9}{12} = \frac{19}{12}$ $= 1\frac{7}{12}$. Never add the denominators.

9.4 Multiplying fractions and finding a fraction of an amount

Multiply numerators together and denominators together. Cancel common factors across the product before multiplying to keep the numbers small. Convert mixed numbers first. Finding a fraction of an amount means multiplying by that fraction.

Worked example $1\frac{1}{2} \times \frac{4}{9} = \frac{3}{2} \times \frac{4}{9} = \frac{2}{3}$. Also, $\frac{3}{5}$ of 40 $= \frac{3}{5} \times 40 = 24$. Multiplication by a positive proper fraction reduces a positive amount.

9.5 Dividing fractions and reciprocals

The reciprocal of a nonzero number has product 1 with that number. To divide by a nonzero fraction, multiply by its reciprocal. Invert only the divisor. A whole number can be written over 1. Zero has no reciprocal and cannot be a divisor.

Worked example $2\frac{1}{4} \div \frac{3}{8} = \frac{9}{4} \times \frac{8}{3} = 6$. This asks how many groups of three eighths fit into two and one quarter.

9.6 Mixed operations and word problems

Apply brackets and the usual operation hierarchy. Convert mixed numbers before products or quotients, and keep exact fractions until the end. In a word problem, identify the whole. A fraction of the remainder uses a new whole after the first action.

Worked example A tank holds 60 L. First $\frac{1}{3}$ is used, leaving 40 L. Then $\frac{1}{4}$ of the remainder is used: $\frac{1}{4} \times 40 = 10$ L. The final amount is 30 L, not 25 L.

Common mistake to avoid

Cancellation is allowed between factors in a product. You cannot cancel terms across addition or subtraction without first factoring the whole expression.

Chapter 9 practice

Work in your notebook. Show your method and include units where needed.

1. Simplify $\frac{42}{56}$.

2. Calculate $\frac{5}{6} + \frac{3}{8}$.

3. Calculate $3\frac{1}{2} - 1\frac{5}{6}$.

4. Calculate $1\frac{2}{5} \div \frac{7}{10}$.

5. A rope is $4\frac{1}{2}$ m long. Cut off $\frac{2}{3}$ of it. How much remains?

Answers with methods

1. Divide numerator and denominator by 14 to get $\frac{3}{4}$.

2. $\frac{20}{24} + \frac{9}{24} = \frac{29}{24} = 1\frac{5}{24}$.

3. $\frac{7}{2} - \frac{11}{6} = \frac{21}{6} - \frac{11}{6} = \frac{10}{6} = 1\frac{2}{3}$.

4. $\frac{7}{5} \times \frac{10}{7} = 2$.

5. The cut is $\frac{2}{3} \times \frac{9}{2} = 3$ m. Remaining length is $4\frac{1}{2} - 3 = 1\frac{1}{2}$ m.

Chapter 10

Decimals fractions and percentages

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

10.1 Decimal notation and calculation

Decimals write fractions with denominators 10, 100, 1,000 and so on. Align place values for addition and subtraction. Trailing zeros do not change a value. Multiplication and division by powers of ten change place values rather than simply adding or removing zeros.

Worked example 4.08 + 0.7 = 4.08 + 0.70 = 4.78. Also, 3.47 × 100 = 347 and 3.47 ÷ 100 = 0.0347.

10.2 Converting fractions to decimals

Divide the numerator by the denominator. A simplified fraction has a terminating decimal exactly when its denominator has no prime factors except 2 and 5. Other rational fractions repeat. Use a recurring mark or an ellipsis to show that the pattern continues.

Worked example $\frac{3}{8}$ = 3 ÷ 8 = 0.375. The denominator 8 = 2^3 gives a terminating decimal. In contrast, $\frac{2}{3}$ = 0.666… repeats forever.

10.3 Converting decimals to fractions

Read the decimal as a number of tenths, hundredths or thousandths, then simplify. Preserve leading decimal zeros when choosing the denominator. For a repeating decimal, an algebraic subtraction can remove the repeating part.

Worked example 0.045 = $\frac{45}{1000}$ = $\frac{9}{200}$. For x = 0.777…, 10x = 7.777…, so 9x = 7 and x = $\frac{7}{9}$.

10.4 Percentages and equivalent forms

Percent means per hundred. Convert a decimal to a percentage by multiplying by 100. Convert a percentage to a decimal by dividing by 100. To convert a fraction to a percentage, multiply it by 100%. Percentages greater than 100% describe more than one whole.

Worked example $\frac{3}{5} = 0.6 = 60\%$. Also, $7\% = 0.07 = \frac{7}{100}$, while $125\% = 1.25 = 1\frac{1}{4}$.

10.5 Choosing useful representations

Use the form that makes a calculation easiest. Exact common fractions can simplify mental arithmetic. Retain recurring fractions exactly when an exact answer is needed, and round only when instructed. Know common equivalents such as one half, one quarter, three quarters and one fifth.

Worked example $\frac{1}{2} = 0.5 = 50\%$; $\frac{1}{4} = 0.25 = 25\%$; $\frac{3}{4} = 0.75 = 75\%$; $\frac{1}{5} = 0.2 = 20\%$. To find 25% of 88, calculate $88 \div 4 = 22$.

Chapter 10 practice

Work in your notebook. Show your method and include units where needed.

1. Convert $\frac{7}{20}$ to a decimal and percentage.
2. Write 0.125 as a simplified fraction.
3. Write 2.4% as a decimal.
4. Which is greatest: $\frac{2}{5}$, 0.45 or 42%?
5. Find 12.5% of 96.

Answers with methods

1. $\frac{7}{20} = \frac{35}{100} = 0.35 = 35\%$.

2. $\frac{125}{1000} = \frac{1}{8}$.

3. $2.4 \div 100 = 0.024$.

4. They are 0.40, 0.45 and 0.42, so 0.45 is greatest.

5. $12.5\% = \frac{1}{8}$, so $96 \div 8 = 12$.

Common mistake to avoid

0.5% means 0.005 of an amount. It is not the same as 0.5, which is 50%.

Chapter 11

Rounding estimation and significant figures

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

11.1 Rounding to a place value

Choose the place you are rounding to, then look at the next digit. If that digit is 5 or more, increase the retained digit by 1. Otherwise keep it. Replace later whole number places with zeros and remove unnecessary later decimal places. This course uses the usual school rounding convention.

Worked example 47,682 to the nearest thousand is 48,000 because the hundreds digit is 6. To the nearest hundred it is 47,700 because the tens digit is 8.

11.2 Decimal places

Decimal places count digits after the decimal point, including zeros. Round by examining the next place. When the requested accuracy needs a trailing zero, retain it to show precision. Carrying can change digits farther to the left.

Worked example 6.295 to two decimal places is 6.30. The third decimal digit is 5, so the hundredths increase from 9 and create a carry. The final zero shows two decimal places.

11.3 Significant figures

Count significant figures from the first nonzero digit. Zeros between significant digits count. Leading zeros do not count. Trailing zeros after a decimal point count when they show stated precision. Use scientific notation when whole number trailing zeros would be ambiguous.

Worked example 0.004072 to three significant figures is 0.00407. The significant digits begin at 4. The number 6.20 has three significant figures, while 6.2 has two.

11.4 Estimating calculations

Replace values with nearby easy numbers before calculating. An estimate checks whether an exact result has a sensible size. One significant figure often gives a quick estimate. Use $\approx$ for approximately equal, because rounding changes the original values.

Worked example $\dfrac{198 \times 4.9}{0.51} \approx \dfrac{200 \times 5}{0.5} = 2{,}000$. An exact calculator result near 2,000 is plausible; a result near 20 is not.

11.5 Accuracy and bounds

A rounded measurement represents an interval. For positive values rounded to the nearest unit, the lower bound is half a unit below and the upper bound is half a unit above. Under ordinary school conventions, include the lower bound and exclude the upper bound. State the unit and rounding step.

Worked example A length recorded as 12.4 cm to the nearest 0.1 cm satisfies $12.35 \text{ cm} \leq \text{length} < 12.45 \text{ cm}$. The half step is 0.05 cm. A length of 12.45 cm would round to 12.5 cm.

Chapter 11 practice

Work in your notebook. Show your method and include units where needed.

1. Round 38,749 to the nearest hundred.
2. Round 8.976 to two decimal places.
3. Round 0.006384 to two significant figures.
4. Estimate 49.8×19.7.
5. A mass is 27 g to the nearest gram. State its bounds.

Answers with methods

1. 38,700 because the tens digit is 4.

2. 8.98 because the third decimal digit is 6.

3. 0.0064 because the first two significant digits are 6 and 3, followed by 8.

4. $50 \times 20 = 1{,}000$, so the result should be close to 1,000.

5. $26.5 \text{ g} \leq \text{mass} < 27.5 \text{ g}$.

Common mistake to avoid

Decimal places and significant figures are different. 0.00456 has five decimal places but only three significant figures.

Chapter 12

Ratio proportion and the unitary method

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

12.1 Ratio notation and simplification

A ratio compares amounts in a stated order. Write quantities in the same units before simplifying. Divide every part by the same common factor. A ratio compares parts with parts; a fraction can compare one part with the total. A ratio need not add to 1 or 100.

Worked example Red to blue beads is 12 : 18 = 2 : 3. There are 5 parts in total, so the fraction that is red is $\frac{2}{5}$, not $\frac{2}{3}$. Also, 1 m : 25 cm = 100 : 25 = 4 : 1.

12.2 Sharing in a ratio

Add the ratio parts, divide the total by that number, then multiply by each part. Check that the shares add to the original amount. For a difference problem, compare the difference in parts with the given difference in amounts.

Worked example Share 360 in the ratio 2 : 3 : 4. There are 9 parts, so each part is 40. The shares are 80, 120 and 160. Their sum is 360.

12.3 The unitary method

Find the value of one unit and then scale to the required number. This method works when the rate is constant. Include units at each step to avoid reversing the calculation. The same method applies to prices, recipes, wages and distances at constant speed.

Worked example Five notebooks cost 175 rupees. One notebook costs 175 ÷ 5 = 35 rupees. Eight notebooks cost 8 × 35 = 280 rupees, assuming the price per notebook stays the same.

12.4 Direct proportion

Two quantities are directly proportional when their ratio remains constant. Multiplying one by a factor multiplies the other by the same factor. Write $y = kx$, where k is the constant of proportionality. A direct proportion graph is a straight line through the origin.

Worked example A recipe uses 180 g flour for 6 cakes. For 15 cakes, the scale factor is $15 \div 6 = 2.5$, so flour needed is $180 \times 2.5 = 450$ g.

12.5 Inverse proportion

Two positive quantities are inversely proportional when their product remains constant. Increasing one decreases the other by the same factor. This model requires unchanged total work and a constant rate per worker or machine. It does not apply automatically to every real situation.

Worked example Four equally efficient machines finish a job in 9 hours. The work is $4 \times 9 = 36$ machine hours. Six such machines take $36 \div 6 = 6$ hours, assuming they can work together without delays.

12.6 Scale drawings

A scale compares a drawing length with the corresponding real length. Convert both to the same units. A scale of $1 : n$ means 1 unit on the drawing represents n of the same units in reality. The scale factor for area is n^2 and for volume is n^3.

Worked example On a $1 : 50{,}000$ map, 3 cm represents 150,000 cm = 1.5 km. A real distance of 2 km is 200,000 cm, so it measures 4 cm on this map.

Chapter 12 practice

Work in your notebook. Show your method and include units where needed.

1. Simplify 45 : 60.

2. Share 420 rupees in the ratio 3 : 4.

3. Three kilograms of rice cost 156 rupees. Find the cost of 5 kg.

4. Eight workers need 15 days. How long do 12 equally efficient workers need?

5. On a 1 : 20,000 map, a road is 7 cm long. Find its real length.

Answers with methods

1. Divide both terms by 15 to obtain 3 : 4.

2. 7 parts give 60 rupees per part. The shares are 180 and 240 rupees.

3. One kilogram costs 52 rupees, so 5 kg cost 260 rupees.

4. 8 × 15 = 120 worker days. Time = 120 ÷ 12 = 10 days, assuming constant efficiency and divisible work.

5. 7 × 20,000 = 140,000 cm = 1.4 km.

Common mistake to avoid

Inverse proportion uses a constant product. Direct proportion uses a constant ratio. Decide which relationship the context supports before calculating.

Chapter 13

Percentages profit loss discounts and tax

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

13.1 Finding a percentage of an amount

A percentage is a fraction out of 100. Multiply the amount by the percentage divided by 100. For mental calculations, combine easy percentages such as 10%, 5% and 1%. Always identify the amount that represents the original whole.

Worked example 15% of 240 = $\frac{15}{100}$ × 240 = 36. Mentally, 10% is 24 and 5% is 12, giving 36. To express 36 as a percentage of 240, calculate $\frac{36}{240}$ × 100% = 15%.

13.2 Percentage increase and decrease

For an increase of p%, multiply by 1 + p ÷ 100. For a decrease, multiply by 1 − p ÷ 100. Percentage change compares the change with the original value. Equal percentage increases and decreases usually do not cancel because the base changes.

Worked example A price of 800 rises by 12%, giving 800 × 1.12 = 896. A later decrease of 12% gives 896 × 0.88 = 788.48, not 800.

13.3 Profit and loss

Profit = selling price − cost price. Loss = cost price − selling price. In standard school problems, profit or loss percentage uses cost price as the base. Include additional stated costs before comparing with selling price. A percentage of selling price is a different calculation.

Worked example An item costs 500 rupees and sells for 575 rupees. Profit is 75 rupees. Profit percentage = $\frac{75}{500} \times 100\% = 15\%$. If it sells for 450 rupees, the loss is 50 rupees or 10%.

13.4 Discounts and tax

A discount reduces a marked price. A tax increases the applicable price. Apply successive changes in the order specified. In the examples here, tax applies after the discount. Rates in exercises are hypothetical and are not a statement of current tax rules.

Worked example An item marked 1,200 rupees has 20% off, then 5% tax. Discounted price = 1,200 × 0.80 = 960. Tax = 960 × 0.05 = 48. Final price = 1,008 rupees.

13.5 Reverse percentages

To recover an original value, divide the final value by the multiplier. Do not subtract the same percentage from the final amount because it uses a different base. State whether the final value includes an increase or a decrease.

Worked example A price after a 15% discount is 680 rupees. This is 85% of the original, so original price = 680 ÷ 0.85 = 800 rupees. Check: 800 × 0.85 = 680.

13.6 Simple interest

Simple interest uses only the original principal. Interest = principal × annual rate × time, when the rate is a decimal and time is in years. Total amount equals principal plus interest. This differs from compound interest, which also earns interest on earlier interest.

Worked example On 2,000 rupees at 6% simple interest per year for 3 years, interest = 2,000 × 0.06 × 3 = 360 rupees. The total is 2,360 rupees.

Chapter 13 practice

Work in your notebook. Show your method and include units where needed.

1. Find 18% of 350.
2. Increase 640 by 25%.
3. An item costs 750 and sells for 690. Find the loss percentage.
4. A 900 rupee item has 10% off, then 8% tax. Find the final price.
5. A value after a 20% increase is 540. Find the original.

Answers with methods

1. $\frac{18}{100} \times 350 = 63$.

2. $640 \times 1.25 = 800$.

3. Loss = 60. Loss percentage = $\frac{60}{750} \times 100\% = 8\%$.

4. $900 \times 0.90 \times 1.08 = 874.80$ rupees.

5. $540 \div 1.20 = 450$.

Common mistake to avoid

A discount percentage uses the marked price. A profit percentage usually uses cost price. Label the base before inserting values.

Chapter 14

Squares cubes roots and indices

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

14.1 Squares and square roots

The square of a number is its product with itself. The principal square root of a nonnegative number is the nonnegative number whose square gives it. Memorise small square numbers to recognise them quickly. An equation such as $x^2 = 25$ has two real solutions even though $\sqrt{25}$ has one principal value.

Worked example $7^2 = 49$ and $\sqrt{49} = 7$. The solutions to $x^2 = 49$ are $x = 7$ and $x = -7$ because both numbers have square 49.

14.2 Cubes and cube roots

A cube multiplies a number by itself three times. The cube root reverses cubing. A negative number has a negative cube and a negative real cube root. Square units measure area; cubic units measure volume.

Worked example $4^3 = 64$, so $\sqrt[3]{64} = 4$. Also, $(-3)^3 = -27$ and $\sqrt[3]{(-27)} = -3$. A cube of side 4 cm has volume 64 cm^3.

14.3 Products and quotients of powers

An index tells how often the base appears as a factor. For the same nonzero base, multiply powers by adding indices and divide by subtracting indices. The bases must match. These rules describe products and quotients, not sums.

Worked example $2^3 \times 2^4 = 2^7 = 128$. Also, $5^6 \div 5^2 = 5^4 = 625$. But $2^3 + 2^4 = 8 + 16 = 24$, not 2^7.

14.4 Powers of powers and products

Raise a power to another power by multiplying the indices. A power of a product applies to each factor. A power of a quotient applies to numerator and denominator, provided the denominator is not zero. Brackets define the base.

Worked example $(3^2)^3 = 3^6 = 729$. Also, $(2 \times 5)^2 = 2^2 \times 5^2 = 100$, and $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$.

14.5 Zero negative and fractional indices

For a nonzero base a, $a^0 = 1$ and a raised to a negative integer is the reciprocal of the corresponding positive power. For positive a, a to the power one half means $\sqrt{a}$ and one third means $\sqrt[3]{a}$. Zero to a negative power is undefined.

Worked example $7^0 = 1$. Also, $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$. The value 16 raised to the power $\frac{1}{2}$ is $\sqrt{16} = 4$. The expression 0^0 is not assigned a value in this course.

Chapter 14 practice

Work in your notebook. Show your method and include units where needed.

1. Calculate 13^2.
2. Find $\sqrt[3]{125}$.
3. Simplify $3^2 \times 3^5$ as one power.
4. Evaluate 10^{-2}.
5. Solve $x^2 = 36$.

Answers with methods

1. $13 \times 13 = 169$.

2. 5, because $5^3 = 125$.

3. 3^7, because $2 + 5 = 7$.

4. $\frac{1}{10^2} = \frac{1}{100} = 0.01$.

5. $x = 6$ or $x = -6$. Both square to 36.

Common mistake to avoid

$\sqrt{(a + b)}$ is generally not $\sqrt{a} + \sqrt{b}$. For example, $\sqrt{(9 + 16)} = 5$, but $\sqrt{9} + \sqrt{16} = 7$.

Chapter 15

Scientific notation

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

15.1 The standard form convention

Scientific notation writes a nonzero number as $a \times 10^n$, where $1 \le |a| < 10$ and n is an integer. For positive numbers, $1 \le a < 10$. It makes very large and very small values easier to compare and calculate. The coefficient must satisfy the convention.

Worked example $4{,}700{,}000 = 4.7 \times 10^6$. The expression 47×10^5 has the same value but is not normalised scientific notation because 47 is at least 10.

15.2 Large numbers and positive powers

Count how many places the decimal point moves to obtain a coefficient between 1 and 10. For numbers of at least 10, the exponent is positive. Check by expanding the power of ten. Keep the original significant digits unless rounding is requested.

Worked example $62{,}300 = 6.23 \times 10^4$ because $6.23 \times 10{,}000 = 62{,}300$. A population of 8,140,000 can be written 8.14×10^6.

15.3 Small numbers and negative powers

A positive number smaller than 1 has a negative exponent in scientific notation. A negative exponent means repeated division by ten, not a negative value. Leading zeros help count place shifts but are not significant figures.

Worked example $0.000508 = 5.08 \times 10^{-4}$. Expanding gives $5.08 \div 10{,}000 = 0.000508$. The value remains positive even though its exponent is negative.

15.4 Multiplying and dividing

Multiply or divide coefficients separately and use index laws for powers of ten. Normalise the result afterwards. Include units if the calculation represents a measurement. Check the order of magnitude to catch exponent errors.

Worked example $(3 \times 10^5)(4 \times 10^{-2}) = 12 \times 10^3 = 1.2 \times 10^4$. Also, $(8 \times 10^7) \div (2 \times 10^3) = 4 \times 10^4$.

15.5 Adding subtracting and comparing

To add or subtract, express both terms with the same power of ten first. Then combine coefficients and normalise. For positive normalised values, compare exponents first and coefficients second. Round the coefficient only after completing a calculation.

Worked example $5.2 \times 10^6 + 7 \times 10^5 = 5.2 \times 10^6 + 0.7 \times 10^6 = 5.9 \times 10^6$. Also, 2.1×10^4 is greater than 9.8×10^3.

Chapter 15 practice

Work in your notebook. Show your method and include units where needed.

1. Write 92,000 in scientific notation.
2. Write 0.000063 in scientific notation.
3. Expand 4.08×10^3.
4. Calculate $(6 \times 10^4) \div (3 \times 10^{-2})$.
5. Calculate $3.4 \times 10^5 + 2 \times 10^4$.

Answers with methods

1. 9.2×10^4.

2. 6.3×10^{-5}.

3. $4.08 \times 1{,}000 = 4{,}080$.

4. 2×10^6, because $4 - (-2) = 6$.

5. $3.4 \times 10^5 + 0.2 \times 10^5 = 3.6 \times 10^5$.

Common mistake to avoid

Adding exponents is a multiplication rule. It does not apply when adding two scientific notation values.

Chapter 16

Measurement units and conversions

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

16.1 Choosing units and measuring

A measurement combines a number with a unit. Choose a unit appropriate to the size and quantity. Length, mass, capacity, area and volume measure different attributes. Read a scale by finding the value between labelled marks, counting intervals rather than tick marks, and viewing straight on.

Worked example A ruler interval from 2 cm to 3 cm split into ten equal intervals has 1 mm per interval. An object ending at the sixth small mark after 2 cm measures 2.6 cm or 26 mm if it starts at zero.

16.2 Length and mass conversions

For metric length, 10 mm = 1 cm, 100 cm = 1 m and 1,000 m = 1 km. For mass, 1,000 mg = 1 g, 1,000 g = 1 kg and 1,000 kg = 1 tonne. Multiply to express a measurement in smaller units and divide to use larger units.

Worked example 3.75 km = 3,750 m. Also, 4,250 g = 4.25 kg. A combined mass of 2 kg 350 g and 850 g is 2,350 g + 850 g = 3,200 g = 3.2 kg.

16.3 Capacity and volume conversions

Capacity describes how much a container can hold. Volume describes the space occupied. One litre equals 1,000 millilitres, and 1 mL equals 1 cm^3. One litre also equals 1,000 cm^3. Keep the quantity type clear when choosing units.

Worked example 2.4 L = 2,400 mL = 2,400 cm^3. A bottle containing 750 mL has 0.75 L. Three such bottles hold 2.25 L.

16.4 Area and cubic unit conversions

Square the length conversion factor for area and cube it for volume. A square metre contains 100 × 100 square centimetres. A cubic metre contains 100 × 100 × 100 cubic centimetres. One hectare is 10,000 m².

Worked example 1 m² = 10,000 cm², so 2.3 m² = 23,000 cm². Also, 1 m³ = 1,000,000 cm³ = 1,000 L. Therefore, 0.04 m³ = 40 L.

16.5 Combining measurements and accuracy

Convert measurements to compatible units before adding or comparing. State the final unit and sensible accuracy. Measuring instruments have limited resolution, so avoid giving more precision than the data supports. Distinguish an exact unit conversion from an approximate measured value.

Worked example A ribbon is 1.8 m long and another is 65 cm. Convert 1.8 m to 180 cm, then add: 180 + 65 = 245 cm = 2.45 m. The conversion is exact.

Chapter 16 practice

Work in your notebook. Show your method and include units where needed.

1. Convert 6.08 m to centimetres.
2. Convert 3,450 mg to grams.
3. Convert 1.25 L to millilitres.
4. Convert 72,000 cm² to m².
5. Find 2 m 35 cm − 87 cm.

Answers with methods

1. 6.08 × 100 = 608 cm.

2. 3,450 ÷ 1,000 = 3.45 g.

3. 1.25 × 1,000 = 1,250 mL.

4. 72,000 ÷ 10,000 = 7.2 m².

5. 235 − 87 = 148 cm = 1 m 48 cm.

Common mistake to avoid

Converting m² to cm² requires multiplying by 10,000, not 100. The exponent on a unit changes the conversion factor.

Chapter 17

Time money rates speed and distance

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

17.1 Time units and clock notation

One minute has 60 seconds and one hour has 60 minutes. A day has 24 hours. In 24 hour notation, afternoon hours run from 13 to 23. Midnight is 00:00 and noon is 12:00. Convert minutes to a fraction of an hour before using hourly rates.

Worked example 2 hours 45 minutes = $2 + \frac{45}{60}$ = 2.75 hours. The time 7:25 pm is 19:25, while 07:25 is 7:25 am.

17.2 Elapsed time and timetables

Find elapsed time by counting through convenient points such as the next hour or midnight. Include any stops or waiting time specified. If an arrival is on the next day, add the time remaining to midnight to the time after midnight.

Worked example A journey starts at 22:40 and ends at 01:15 the next day. From 22:40 to midnight is 1 hour 20 minutes. Add 1 hour 15 minutes to obtain 2 hours 35 minutes.

17.3 Money budgets and change

Write money to the appropriate currency precision, usually two decimal places for rupees and paise. Total the costs before finding change or remaining budget. Use rates provided in a question for currency conversion; real exchange rates vary. Round money at the final stated stage.

Worked example Four books at 68.50 rupees each cost 274.00 rupees. Change from 500.00 rupees is 226.00 rupees. At a stated rate of 1 unit = 80 rupees, 25 units are worth 2,000 rupees.

17.4 Rates and best value

A rate compares different units, such as rupees per kilogram or litres per minute. Divide by the number of units to find a unit rate. Compare unit prices only when quality and relevant conditions are comparable. Constant rates allow direct scaling.

Worked example A 3 kg pack costs 216 rupees, so its unit price is 72 rupees per kg. A 5 kg pack costs 350 rupees, so it costs 70 rupees per kg and has the lower unit price.

17.5 Speed distance and time

Average speed = distance ÷ elapsed time. Rearranging gives distance = speed × time and time = distance ÷ speed. Use compatible units. To convert m/s to km/h multiply by 3.6; to convert km/h to m/s divide by 3.6. Average speed for a whole trip uses total distance and total time.

Worked example A car travels 150 km in 2.5 hours. Average speed = 150 ÷ 2.5 = 60 km/h. A speed of 72 km/h equals 72 ÷ 3.6 = 20 m/s.

Chapter 17 practice

Work in your notebook. Show your method and include units where needed.

1. Convert 3 hours 18 minutes to hours.

2. Find the elapsed time from 09:48 to 12:15.

3. Six pens cost 17.50 rupees each. Find change from 200 rupees.

4. A cyclist travels at 18 km/h for 40 minutes. Find the distance.

5. A trip covers 60 km in 1 hour and 40 km in 1 hour. Find average speed.

Answers with methods

1. $3 + \frac{18}{60}$ = 3.3 hours.

2. 12 minutes to 10:00, 2 hours to 12:00, then 15 minutes: 2 hours 27 minutes.

3. Cost = 105 rupees. Change = 95 rupees.

4. 40 minutes = $\frac{2}{3}$ hour. Distance = $18 \times \frac{2}{3}$ = 12 km.

5. Total distance 100 km divided by total time 2 hours gives 50 km/h.

Common mistake to avoid

1 hour 30 minutes is 1.5 hours, not 1.30 hours. The hour uses 60 minutes, not 100.

Chapter 18

Basic algebra

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

18.1 Variables expressions and equations

A variable represents a number that may change or be unknown. An expression has terms connected by operations. An equation states that two expressions are equal. In $4x + 7$, x is the variable, 4 is its coefficient and 7 is a constant. Multiplication between a number and a variable is usually unwritten.

Worked example If x is the price of one ticket, $3x + 20$ represents the price of three tickets plus a 20 rupee booking fee. The statement $3x + 20 = 170$ is an equation.

18.2 Collecting like terms

Like terms have exactly the same variable part, including powers. Add or subtract their coefficients. Constants combine with constants. Unlike terms remain separate. Remember that x means $1x$ and $-x$ means $-1x$.

Worked example $5x + 3y - 2x + 4 = 3x + 3y + 4$. In contrast, $2x + 3x^2$ cannot be combined because x and x^2 are different variable parts.

18.3 Substitution and evaluating formulae

Replace each variable with its stated value and use brackets around negative values. Then apply the operation hierarchy. Repeated letters must receive the same value throughout. Include units when a formula represents a measurement.

Worked example Evaluate $2a^2 - 3b$ when $a = -2$ and $b = 5$. The result is $2(-2)^2 - 3(5) = 8 - 15 = -7$.

18.4 Expanding and factorising

Expand a bracket by multiplying every term inside by the outside factor. A negative factor changes the sign of each term. Factorising reverses expansion by taking out a common factor. Check by multiplying back.

Worked example $3(2x - 5) = 6x - 15$. Also, $-2(x + 4) = -2x - 8$. The expression $12x + 18$ factorises as $6(2x + 3)$.

18.5 Solving simple equations

An equation stays balanced when you perform the same valid operation on both sides. Undo operations in reverse order. For variables on both sides, collect variable terms on one side and constants on the other. Check the solution in the original equation.

Worked example Solve $3x + 7 = 22$. Subtract 7 to get $3x = 15$, then divide by 3 to get $x = 5$. Check: $3 \times 5 + 7 = 22$. For $5x - 4 = 2x + 11$, subtract $2x$ and add 4 to get $3x = 15$.

18.6 Translating word problems

Choose a variable and say what it means. Translate the relationships into an equation before solving. Consecutive integers can be written n, $n + 1$ and $n + 2$. State the final answer in words and check it fits the situation.

Worked example A number doubled and then increased by 9 gives 31. Let the number be n. Then $2n + 9 = 31$, so $2n = 22$ and $n = 11$. Check: $22 + 9 = 31$.

Chapter 18 practice

Work in your notebook. Show your method and include units where needed.

1. Simplify $7a - 2a + 4 - 9$.
2. Evaluate $3x + 2y$ when $x = -4$ and $y = 7$.
3. Expand $4(2x - 3)$.
4. Solve $5x - 6 = 29$.
5. Three consecutive integers total 48. Find them.

Answers with methods

1. $5a - 5$.

2. $3(-4) + 2(7) = -12 + 14 = 2$.

3. $8x - 12$.

4. Add 6: $5x = 35$. Divide by 5: $x = 7$.

5. $n + (n + 1) + (n + 2) = 48$, so $3n + 3 = 48$ and $n = 15$. The integers are 15, 16 and 17.

Common mistake to avoid

$3(x + 2)$ means $3x + 6$. Multiplying only the first term inside the bracket gives an incorrect expression.

Chapter 19

Number patterns and sequences

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

19.1 Recognising a sequence

A sequence is an ordered list of terms. Each term has a position, beginning with position 1 unless stated otherwise. Look for a rule involving differences, ratios, alternating operations or known number families. A few terms can fit many rules, so describe the intended rule clearly.

Worked example The sequence 5, 9, 13, 17 has constant difference 4. Its fifth term is 21. The number 5 is a term value, while 1 is its position.

19.2 Arithmetic sequences

An arithmetic sequence has a constant difference d. Its nth term is first term + (n − 1)d. Simplify this expression when useful. The rule gives any term directly without listing all earlier terms. A negative common difference gives a decreasing sequence.

Worked example For 7, 10, 13, 16, the first term is 7 and d = 3. The nth term is $7 + 3(n - 1) = 3n + 4$. The twentieth term is $3 \times 20 + 4 = 64$.

19.3 Geometric sequences

A geometric sequence has a constant multiplying factor, called the common ratio. Its nth term is first term × ratio raised to the power n − 1. The ratio can be a fraction or negative. Check the ratio rather than assuming every growing pattern is arithmetic.

Worked example The sequence 3, 6, 12, 24 has ratio 2. Its nth term is $3 \times 2^{n-1}$. The sixth term is $3 \times 2^5 = 96$. The sequence 80, 40, 20, 10 has ratio $\frac{1}{2}$.

19.4 Square cubic triangular and recursive patterns

Square numbers are 1, 4, 9, 16 and so on. Cubes begin 1, 8, 27, 64. Triangular numbers begin 1, 3, 6, 10 and add consecutive counting numbers. A recursive rule defines a term using earlier terms, so starting values must be given.

Worked example A sequence beginning 1, 1 and adding the previous two terms continues 2, 3, 5, 8, 13. Triangular numbers have nth term $\frac{n(n+1)}{2}$, so the sixth is $\frac{6 \times 7}{2} = 21$.

19.5 Testing whether a value is a term

Set the nth term expression equal to the proposed value and solve for n. A term exists only if the resulting position is a positive integer under the stated indexing. Use a pattern rule to explain rather than relying on a long list.

Worked example Is 100 in the sequence with nth term 4n + 1? Solve 4n + 1 = 100 to get n = 24.75, so no. For 101, n = 25, so it is the twenty fifth term.

Chapter 19 practice

Work in your notebook. Show your method and include units where needed.

1. Continue 18, 14, 10, 6 for two terms.
2. Find the nth term of 6, 11, 16, 21.
3. Find the tenth term of 2n + 7.
4. Continue 81, 27, 9, 3 for two terms.
5. Is 62 in the sequence 5n + 2?

Answers with methods

1. The difference is −4, so the next terms are 2 and −2.

2. 6 + 5(n − 1) = 5n + 1.

3. 2 × 10 + 7 = 27.

4. Divide by 3 each time, giving 1 and $\frac{1}{3}$.

5. 5n + 2 = 62 gives n = 12, so yes, it is the twelfth term.

Common mistake to avoid

The common difference is not usually the whole nth term rule. For 5, 8, 11, the rule is 3n + 2, not just 3n.

Chapter 20

Lines angles and geometric shapes

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

20.1 Points lines rays and segments

A point marks a position. A line extends endlessly in both directions; a ray has one endpoint; a segment has two endpoints. Parallel lines in a plane never meet. Perpendicular lines meet at a right angle. An angle measures a turn, usually in degrees.

Worked example In a rectangle, opposite sides are parallel and adjacent sides are perpendicular. A right angle measures 90°. A full turn measures 360° and a half turn 180°.

20.2 Types of angles and measuring

An acute angle is between 0° and 90°. A right angle is 90°. An obtuse angle is between 90° and 180°. A straight angle is 180° and a reflex angle is between 180° and 360°. To use a protractor, place its centre on the vertex and its baseline along one arm, then read from zero on that arm.

Worked example If an angle opens wider than a right angle but less than a straight angle, it is obtuse. An angle of 235° is reflex. Its smaller remaining angle around the point is $360° - 235° = 125°$.

20.3 Angle relationships

Angles on a straight line total 180° and angles around a point total 360°. Vertically opposite angles are equal. For parallel lines cut by a transversal, corresponding angles are equal, alternate angles are equal, and interior angles on the same side total 180°. State the reason for each step.

Worked example Two adjacent angles on a straight line are x and 68°. Therefore $x = 180° - 68° = 112°$. If a corresponding angle lies on a parallel line, it also measures 68°.

20.4 Triangles and quadrilaterals

A triangle has angle sum 180°. An equilateral triangle has three equal sides and three 60° angles. An isosceles triangle has equal base angles. A quadrilateral has angle sum 360°. Rectangles have four right angles; squares also have four equal sides. A rhombus has four equal sides; a parallelogram has two pairs of parallel sides.

Worked example A triangle has angles 48° and 67°, so the third is 180° − 48° − 67° = 65°. A square is also a rectangle, rhombus and parallelogram because it satisfies each definition.

20.5 Polygons circles and symmetry

An n sided polygon has interior angle sum (n − 2) × 180°. In a regular polygon all sides and all interior angles are equal, and each exterior angle is 360° ÷ n. A circle has radius from centre to circumference and diameter twice the radius. A line of symmetry reflects a shape onto itself; rotational symmetry counts matching positions in a full turn.

Worked example A regular hexagon has each exterior angle 360° ÷ 6 = 60° and each interior angle 120°. A square has four lines of symmetry and rotational symmetry of order 4. A circle of radius 5 cm has diameter 10 cm.

20.6 Solids and nets

A solid has three dimensions. A polyhedron has flat polygonal faces, straight edges and vertices. A net unfolds a solid into a flat arrangement of its surfaces. A cube has 6 square faces, 12 edges and 8 vertices. A cylinder has two circular flat faces and one curved surface.

Worked example A cuboid net contains six rectangles, arranged so that folding pairs opposite faces. A triangular prism has 5 faces, 9 edges and 6 vertices. Its two end faces are congruent triangles.

Chapter 20 practice

Work in your notebook. Show your method and include units where needed.

1. Find the supplement of 73°.
2. An isosceles triangle has vertex angle 38°. Find each base angle.
3. Find each interior angle of a regular octagon.
4. A circle has diameter 14 cm. Find its radius.
5. How many edges does a cube have?

Answers with methods

1. 180° − 73° = 107°.

2. (180° − 38°) ÷ 2 = 71°.

3. Sum = 6 × 180° = 1,080°. Each angle = 1,080° ÷ 8 = 135°.

4. 14 ÷ 2 = 7 cm.

5. 12 edges. Each joins two vertices and belongs to two faces.

Common mistake to avoid

A diagram may not be drawn to scale. Use stated measurements and geometric reasons rather than judging an angle by its appearance.

Chapter 21

Perimeter area surface area and volume

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

21.1 Perimeter and composite boundaries

Perimeter is the total distance around the outside of a flat shape. Add all boundary lengths in compatible units. A rectangle has perimeter 2(length + width). Find missing sides of composite shapes from equal horizontal or vertical totals. Do not count internal joining lines.

Worked example A rectangle measuring 8 cm by 5 cm has perimeter 2(8 + 5) = 26 cm. If a 2 cm square is cut from one corner, two removed boundary lengths are replaced by two equal new lengths, so the perimeter remains 26 cm.

21.2 Areas of common plane shapes

Area measures the surface inside a boundary and uses square units. Rectangle area is length × width. Triangle area is half × base × perpendicular height. Parallelogram area is base × perpendicular height. Trapezium area is half × sum of parallel sides × perpendicular height.

Worked example A triangle with base 12 cm and perpendicular height 7 cm has area $\frac{1}{2} \times 12 \times 7 = 42$ cm². A trapezium with parallel sides 6 cm and 10 cm and height 4 cm has area $\frac{1}{2} \times (6 + 10) \times 4 = 32$ cm².

21.3 Circles and semicircles

For radius r and diameter d, circumference = 2πr = πd and area = πr². Keep π exact until a rounded answer is required, unless a question gives a value for π. A semicircle has half a circle area, but its perimeter also includes its diameter.

Worked example A semicircle of radius 7 cm has area $\frac{49\pi}{2}$ cm². Its perimeter is $7\pi + 14$ cm. Using $\pi \approx 3.14159$ gives area about 76.97 cm² and perimeter about 35.99 cm.

21.4 Composite area

Split a composite shape into familiar nonoverlapping parts and add their areas. Alternatively, find the area of an enclosing shape and subtract the missing regions. Mark which measurements belong to each part. Areas can be added only when expressed in the same square units.

Worked example An L shape fits inside a 10 cm by 8 cm rectangle with a 4 cm by 3 cm corner missing. Its area is $80 - 12 = 68$ cm². Adding the side lengths would give a perimeter calculation, not an area.

21.5 Surface area

Surface area is the sum of all exposed surfaces of a solid. A cuboid has area $2(lw + lh + wh)$; a cube has area $6s^2$. A closed cylinder has area $2\pi r^2 + 2\pi rh$. An open container may have a missing top, so include only the surfaces present.

Worked example A closed cuboid of length 6 cm, width 4 cm and height 3 cm has surface area $2(24 + 18 + 12) = 108$ cm². Without its 6 cm by 4 cm top, the surface area is $108 - 24 = 84$ cm².

21.6 Volume and capacity

Volume uses cubic units. A cuboid has volume lwh and a cube has volume s^3. A prism has volume cross sectional area × perpendicular length. A cylinder has volume $\pi r^2 h$. Convert volume to capacity using 1 cm³ = 1 mL and 1,000 cm³ = 1 L.

Worked example A triangular prism has cross section area 18 cm² and length 10 cm, so its volume is 180 cm³. A tank measuring 40 cm by 25 cm by 30 cm holds 30,000 cm³ = 30 L when full.

Chapter 21 practice

Work in your notebook. Show your method and include units where needed.

1. Find the area and perimeter of a 9 cm by 4 cm rectangle.
2. Find the area of a triangle with base 15 m and height 8 m.
3. Find the circumference of a circle of radius 6 cm in terms of π.
4. Find the surface area of a cube of side 5 cm.
5. A cuboid is 12 cm by 5 cm by 4 cm. Find its volume.

Answers with methods

1. Area = $9 \times 4 = 36$ cm². Perimeter = $2(9 + 4) = 26$ cm.

2. $\frac{1}{2} \times 15 \times 8 = 60$ m².

3. $2\pi \times 6 = 12\pi$ cm.

4. $6 \times 5^2 = 150$ cm².

5. $12 \times 5 \times 4 = 240$ cm³.

Common mistake to avoid

Surface area uses square units; volume uses cubic units. The sloping side of a triangle is not its perpendicular height unless it is perpendicular to the base.

Chapter 22

Coordinates and simple graphs

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

22.1 The coordinate plane

Two perpendicular axes meet at the origin (0, 0). The horizontal axis is x and the vertical axis is y. An ordered pair (x, y) gives horizontal position first and vertical position second. Choose a clear scale on each axis and keep intervals equal along an axis.

Worked example To plot (−3, 2), move 3 units left from the origin, then 2 units up. The point (2, −3) is different because the coordinates have changed order.

22.2 Quadrants and points on axes

The first quadrant has positive x and y. The second has negative x and positive y. The third has both negative and the fourth has positive x and negative y. A point on an axis is not in any quadrant. On the x axis y = 0, and on the y axis x = 0.

Worked example (4, 5) lies in quadrant I, (−4, 5) in II, (−4, −5) in III and (4, −5) in IV. The point (0, −6) lies on the y axis.

22.3 Horizontal vertical and simple linear graphs

A horizontal line has equation y = constant and a vertical line has equation x = constant. To graph y = mx + c, choose x values, calculate y, plot the pairs and draw the straight line. The value c is the y intercept and m is the gradient.

Worked example For y = 2x + 1, x values −1, 0, 1 and 2 give y values −1, 1, 3 and 5. The points lie on one line, which crosses the y axis at (0, 1).

22.4 Gradient and intercepts

Gradient is vertical change divided by horizontal change. Use two points on the line and subtract coordinates consistently. A positive gradient rises from left to right and a negative one falls. To find an x intercept, set y = 0; for a y intercept, set x = 0.

Worked example Through (1, 3) and (4, 9), gradient = $\frac{9 - 3}{4 - 1} = \frac{6}{3}$ = 2. For y = 2x − 6, the intercepts are (0, −6) and (3, 0).

22.5 Reading graphs in context

Read axis labels, units and scales before interpreting a graph. A point represents a pair of related values. Interpolate within the displayed data range with care; extrapolation beyond it may be unreliable. In a distance time graph, gradient gives speed and a horizontal section means the distance is unchanged.

Worked example A distance time graph rises from (0 h, 0 km) to (2 h, 90 km), giving speed 90 ÷ 2 = 45 km/h. A horizontal section from 2 h to 2.5 h represents a 30 minute stop.

Chapter 22 practice

Work in your notebook. Show your method and include units where needed.

1. In which quadrant is (−5, −2)?
2. Give the equation of the horizontal line through (3, −4).
3. Find y when x = −2 in y = 3x + 5.
4. Find the gradient through (0, 2) and (3, 11).
5. A graph shows 120 km travelled in 3 hours at constant speed. Find the speed.

Answers with methods

1. Quadrant III because both coordinates are negative.

2. y = −4.

3. y = 3(−2) + 5 = −1.

4. $\frac{11 - 2}{3 - 0} = \frac{9}{3} = 3$.

5. 120 ÷ 3 = 40 km/h.

Common mistake to avoid

Coordinates are ordered. Plotting y first swaps the axes and usually gives the wrong point. Label axes and units before plotting.

Chapter 23

Data tables charts averages and range

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

23.1 Collecting and organising data

Data are values or observations collected to answer a question. Categorical data describe groups; numerical data describe quantities. Discrete numerical data are counted, while continuous data are measured. A frequency table records how often each value or category occurs. A fair sample should represent the population being studied.

Worked example Eye colour is categorical, number of siblings is discrete and height is continuous. If scores are 2, 3, 2, 4, 3, 2, the frequencies are 2: three, 3: two and 4: one. The total frequency is 6.

23.2 Bar charts pictograms and line graphs

Bar charts compare categories using separated bars. A pictogram uses a key, and part of a symbol may represent part of the key value. Line graphs display change across an ordered variable such as time. Give a title, labels, units and a consistent scale. Histograms show grouped continuous data with touching bars; unequal class widths require frequency density.

Worked example If one pictogram symbol represents 4 books, three and a half symbols represent 14 books. A bar reaching 25 on a scale increasing by 5 represents 25 items, not five items.

23.3 Pie charts and proportions

A pie chart shows parts of one whole. Sector angle = category frequency divided by total frequency, multiplied by 360°. All sector angles should total 360°. Compare proportions carefully when charts have different totals. A larger sector does not automatically mean a larger count across different groups.

Worked example In a class of 30, 12 choose football. The sector angle is $\frac{12}{30} \times 360° = 144°$. The percentage is $\frac{12}{30} \times 100\% = 40\%$.

23.4 Mean and weighted mean

The mean is the sum of values divided by the number of values. For a frequency table, multiply each value by its frequency, add the products and divide by total frequency. The mean uses every value and can be affected strongly by an extreme value.

Worked example For values 1, 2 and 3 with frequencies 2, 3 and 1, the total is $1 \times 2 + 2 \times 3 + 3 \times 1 = 11$. Total frequency is 6, so mean = $\frac{11}{6} = 1\ \frac{5}{6}$.

23.5 Median mode and range

Order data before finding the median. For an odd count, take the middle value; for an even count, average the middle two. The mode is the most frequent value and may be absent or not unique. Range = maximum − minimum and measures spread.

Worked example For 2, 4, 4, 7, 8, 11, the median is $(4 + 7) \div 2 = 5.5$, the mode is 4 and the range is $11 - 2 = 9$. The mean is $36 \div 6 = 6$.

23.6 Choosing and interpreting summaries

Use an average to describe a typical value and a measure of spread to describe variability. The median can be more representative when an extreme value pulls the mean away from most data. Check sample size and graph scales before drawing a conclusion. A truncated vertical axis can exaggerate differences visually.

Worked example Incomes 20, 22, 23, 25 and 210 have median 23 but mean 60. The unusually large value 210 raises the mean. Reporting the median and range gives more context than the mean alone.

Chapter 23 practice

Work in your notebook. Show your method and include units where needed.

1. Find the mean of 6, 8, 10 and 12.
2. Find the median of 9, 3, 7, 5 and 11.
3. Find the mode and range of 2, 5, 5, 6, 9.
4. A category contains 15 of 60 observations. Find its pie chart angle.
5. Scores 2 and 5 have frequencies 3 and 2. Find the mean.

Answers with methods

1. Sum = 36, count = 4, so mean = 9.

2. Ordered data are 3, 5, 7, 9, 11. The median is 7.

3. Mode = 5. Range = 9 − 2 = 7.

4. $\frac{15}{60} \times 360° = 90°$.

5. $(2 \times 3 + 5 \times 2) \div 5 = 16 \div 5 = 3.2$.

Common mistake to avoid

The mean is not the sum divided by the number of different values. Divide by the total number of observations, including repeated values.

Chapter 24

Basic probability

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

24.1 Outcomes events and sample spaces

An outcome is one possible result of a trial. An event is a collection of outcomes. A sample space lists all possible outcomes once each. A probability model must reflect whether outcomes are equally likely. Words such as certain, likely and impossible describe the chance of an event.

Worked example For a fair six sided die, the sample space is {1, 2, 3, 4, 5, 6}. The event of rolling an even number is {2, 4, 6}. All six single outcomes have equal probability under the fair die model.

24.2 Calculating probability

For equally likely outcomes, probability = number of favourable outcomes divided by total outcomes. Probabilities lie from 0 to 1 inclusive. Zero means impossible and 1 means certain. Give fractions in simplest form, or an equivalent decimal or percentage when requested.

Worked example A bag holds 3 red and 5 blue counters, all equally likely to be selected. P(red) = $\frac{3}{8}$ and P(blue) = $\frac{5}{8}$. The probability of selecting a green counter is 0.

24.3 Complements and mutually exclusive events

The complement of an event means it does not happen. P(not A) = 1 − P(A). Mutually exclusive events cannot occur together in a single trial, so their probabilities can be added. For overlapping events, subtract the overlap to avoid counting it twice.

Worked example On a fair die, P(not 6) = $1 - \frac{1}{6} = \frac{5}{6}$. P(1 or 2) = $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$. For even or greater than 4, favourable outcomes are 2, 4, 5 and 6, giving $\frac{2}{3}$.

24.4 Experimental probability

Relative frequency = observed successes divided by number of trials. It estimates a probability from data. More trials usually make the estimate more stable, but no fixed result is guaranteed. Expected frequency = probability × number of trials and describes a long run expectation.

Worked example A spinner lands on red 46 times in 200 trials. Estimated P(red) = $\frac{46}{200}$ = 0.23. If this estimate is used for 500 trials, the expected red count is 0.23 × 500 = 115.

24.5 Combined independent events

Independent events do not change each other's probabilities. Multiply probabilities along a sequence of independent events. List outcomes systematically using a table or tree when helpful. For draws with replacement, the bag contents return to the original state before the next draw.

Worked example Two fair coin tosses have equally likely outcomes HH, HT, TH and TT. P(two heads) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. P(exactly one head) = $\frac{2}{4} = \frac{1}{2}$.

24.6 Drawing without replacement

Without replacement, the first draw changes the available outcomes, so later probabilities must change. Multiply along each relevant branch and add probabilities of different exclusive routes. State assumptions about random selection.

Worked example A bag contains 3 red and 2 blue counters. Without replacement, P(two red) = $\frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$. With replacement it would be $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$.

Chapter 24 practice

Work in your notebook. Show your method and include units where needed.

1. Find P(rolling a prime) on a fair die.
2. If P(rain) = 0.35, find P(no rain).
3. A bag has 4 green and 6 yellow counters. Find P(green).
4. A fair coin is tossed 120 times. Find the expected number of heads.
5. Two counters are drawn without replacement from 4 red and 1 blue. Find P(two red).

Answers with methods

1. Prime outcomes are 2, 3 and 5. Probability = $\frac{3}{6} = \frac{1}{2}$.

2. $1 - 0.35 = 0.65$.

3. $\frac{4}{10} = \frac{2}{5}$.

4. $120 \times \frac{1}{2} = 60$. The actual number may differ.

5. $\frac{4}{5} \times \frac{3}{4} = \frac{3}{5}$.

Common mistake to avoid

Equally likely outcomes are required for simple counting probability. A spinner with unequal sectors does not give every colour the same chance.

Chapter 25

Word problems reasoning and checking

Learn the ideas below, follow each worked example, then attempt the practice before checking the answers.

25.1 Reading and modelling a problem

Read the whole problem before calculating. Identify what is known, what is required and what the units mean. Choose a diagram, table, bar model or equation to represent the relationships. Operation words give clues, but the meaning of the situation determines the operation.

Worked example A library has 240 books. After donating 35, it buys 58. The model is 240 − 35 + 58, giving 263 books. The final question asks for the new total, so both changes must be included.

25.2 Multi step problems

Break a complex problem into smaller questions. Keep intermediate values exact when possible and label what each one represents. Follow the actual sequence of events. In fraction and percentage problems, decide whether each action uses the original whole or a new remainder.

Worked example A class of 36 has $\frac{2}{3}$ attending a trip. Of those attending, $\frac{3}{4}$ choose lunch. Attending = 24; choosing lunch = 18.

Applying both fractions to 36 separately would answer a different question.

25.3 Working backwards

When a final value is known, reverse each operation in the opposite order. Undo addition with subtraction, multiplication with division, and vice versa. For percentage changes, undo the multiplier by division. Check by running the original process forwards.

Worked example A number is multiplied by 4, then 7 is added, producing 51. Work backwards: 51 − 7 = 44, then 44 ÷ 4 = 11. Forward check: 11 × 4 + 7 = 51.

25.4 Logical reasoning and systematic lists

Use constraints to rule out impossible choices. Make an organised list so cases are neither missed nor repeated. A counterexample disproves a general claim. A few supporting examples alone do not prove a statement for all numbers. Explain the reason behind a pattern when possible.

Worked example To list two digit numbers using 2, 4 and 6 without repetition, organise by tens digit: 24, 26, 42, 46, 62, 64. There are 3 choices for the tens digit and 2 remaining choices for the ones, so 6 numbers.

25.5 Checking reasonableness and exactness

Estimate before calculating. Check with an inverse operation, a different method or substitution. Track units and signs. Decide whether the answer should be a whole number, fraction, decimal or rounded measurement. State any assumption that materially affects the answer.

Worked example A calculation says 48 students need 4 buses holding 15 each. Check capacity: 4 × 15 = 60, while 3 × 15 = 45 is too few. The quotient 48 ÷ 15 = 3.2 must round up for buses.

25.6 Presenting a complete solution

Show the equation or method, meaningful intermediate steps and a final statement with units. Use = only between equal values and ≈ for rounded values. Keep enough precision until the final step. A clear explanation lets another learner follow and check the reasoning.

Worked example A 96 rupee bill is shared equally by 5 friends. Share = 96 ÷ 5 = 19.20 rupees each. Check: 5 × 19.20 = 96. The answer includes the currency and uses two decimal places.

Chapter 25 practice

Work in your notebook. Show your method and include units where needed.

1. A shop has 125 boxes, sells 48 and receives 37. How many remain?
2. A tank has 80 L. After $\frac{3}{8}$ is used, how much remains?
3. A number divided by 3 and then increased by 8 gives 20. Find it.
4. How many 24 seat buses are needed for 130 people?
5. A learner claims every prime is odd. Give a counterexample.

Answers with methods

1. 125 − 48 + 37 = 114 boxes.

2. Used = $\frac{3}{8}$ × 80 = 30 L. Remaining = 50 L.

3. Work backwards: 20 − 8 = 12, then 12 × 3 = 36. Check: 36 ÷ 3 + 8 = 20.

4. 130 ÷ 24 = 5 remainder 10, so 6 buses are required.

5. 2 is prime and even, so the claim is false.

Common mistake to avoid

An answer can be arithmetically correct but unsuitable for the context. You cannot order 3.2 buses or report a negative length for a physical object.